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Zipf zipped

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Abstract

In this paper, I provide a quantitative review of the empirical literature on Zipf's law for cities; the meta-analysis combines 515 estimates from 29 studies. I find that the combined estimate of the Zipf coefficient is significantly larger than 1.0. This finding implies that cities are on average more evenly distributed than suggested by (a strict interpretation of) Zipf's law. I also identify several features that account for differences across the individual point estimates.

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1. Introduction

In 1913, the German geographer Felix Auerbach described an interesting empirical regularity: analyzing the size distribution of cities, he found that the product of the population size of a city and its rank in the distribution appears to be roughly constant for a given territory. Thus, the second-largest city has on average about one-half the population of the largest city, the number 3 city one-third that population, and the number n city $1/n$ that population. Since there is no obvious reason why the hierarchy of cities should follow such a pattern, this rank–size rule, also known as Zipf's law for cities after George Zipf [22], has attracted considerable interest.

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The standard approach to explore city size distributions is simple and intuitive. Based on Auerbach's [1] proposition that $P_i R_i = A$, with P_i the population size of city i , R_i its rank, and A a positive constant, the typical regression takes the form:

$$\ln R_i = \ln A - \alpha \ln P_i.$$

Tests of Zipf's law then include whether this equation describes the city size distribution reasonably well,¹ the estimated coefficient α is close to one,² and A corresponds to the size of the largest city.

While numerous studies have applied this regression for various territories and time periods, the interpretation of the results is ambiguous. Some authors emphasize the strikingly good empirical fit of the log-linear rank-size relationship. Krugman [14, p. 40], for instance, argues that "[w]e are unused to seeing regularities this exact in economics." Gabaix [6, p. 739] notes that "Zipf's law for cities is one of the most conspicuous empirical facts in economics, or in the social sciences generally." Others take a much more skeptical view. Sheppard [19, p. 131], for instance, questions whether the rank-size rule is the best possible representation of city size distributions; other distributional functions may conform more closely to the data. Henderson [10, p. 391] argues that "[i]n general and on average, the rank-size rule simply does not hold."

In this paper, I follow a different approach. Instead of providing another test of Zipf's law, I perform a meta-analysis of estimated Zipf coefficients. That is, I use a set of statistical techniques to combine and evaluate the results from other studies.³ This approach offers several advantages. First, it provides a summary estimate of the parameter of interest, Zipf coefficient α , for a wide range of different data sets and methodologies. Derived from a systematic aggregation of existing research evidence, this estimate appears to be much more credible and accurate than any finding from an individual study. Second, meta-analysis allows to explore the sensitivity of the estimate to characteristics of the underlying study. While deviations from Zipf's law are often attributed to variations in the estimation procedure, only meta-analysis is able to provide a rigorous test of these hypotheses. Finally, a quantitative review of the literature synthesizes the empirical evidence in a rational and unbiased way and thereby comes close to an advice recently given by Gabaix and Ioannides [7]. They argue that most of the controversy on the validity of Zipf's law is misplaced and suggest that "some of the debate on Zipf's law should be cast in terms of how well, or poorly, it fits, rather than whether it can be rejected or not."

¹ The empirical fit of this regression is typically excellent as measured, for instance, by the adjusted R^2 and the statistical significance of the estimated coefficients (most notably α). Some papers do not even report these test statistics. Performance tests therefore occasionally focus on the insignificance of additionally included (e.g., quadratic) terms; Black and Henderson [2] provide a recent application of this test on US data, but see Gabaix and Ioannides [7] for a critique.

² Also deviations from a parameter value of one are of interest; the slope coefficient α is a measure of how evenly distributed is the population, with larger values indicating that the distribution is tilted toward smaller cities.

³ Meta-analysis, a standard tool in medical research, has become increasingly popular in economics. Recent applications include Görg and Strobl [8] on productivity spillovers of multinational companies, Rose [17] on the effect of common currencies on trade, and Disdier and Head [4] on estimates of the distance effect in gravity models. Stanley [21] provides an overview of this technique and further references.

Previewing the main results, I find that the estimates of α are on average significantly larger than 1; the (Zipf) assumption that city size distributions are typically best described by a power law with exponent 1.0 therefore appears to be misplaced.⁴ Aiming to identify features that account for differences across the individual point estimates of α , I find that α estimates are significantly smaller (and then occasionally close to one) if the estimate is based on population data for metropolitan areas (instead of inner cities), the estimate is based on post-1900 data, the estimate is for the size distribution of US cities, the sample comprises only a small number of observations, and the study reports only a single estimation result.

The remainder of the paper is organized as follows. Section 2 discusses the empirical approach and the data. Section 3 presents the results, and Section 4 offers some concluding remarks.

2. Meta-analysis

Meta-analysis begins with a systematic review of the literature and the collection of (all) relevant studies that examine the empirical question of interest. As a general rule, the analysis should cover as many studies as possible: a large number of individual estimates not only improves the precision of the aggregate result; a comprehensive review also increases the probability that the meta-analysis presents a balanced, impartial and otherwise unbiased summary of the existing research evidence.

In order to determine the studies to be included in the analysis, I follow a two-step procedure. In a first step, I perform an EconLit search for the (very general) phrase “city (or cities) and size and distribution” in either the title, the keywords or the abstract of a paper; the search covers the period from 1969 to 2002.⁵ The resulting 269 studies are then checked for an empirical analysis of the rank–size relationship of cities. While most of the references are concerned with market structure and the size distribution of firms, 18 studies contain usable estimates of the Zipf coefficient α .

The selection procedure based on a computer search of a database is very standard in meta-analysis; it helps to avoid selection bias and it is easy to replicate. For my purposes, however, it also has a serious shortcoming: studies completed before 1969, the first year that is covered in the EconLit database, are by design ignored. Since many important contributions in the literature on Zipf’s law were made before that time, this would have been a crucial omission. In a second step, therefore, I also check the references of the selected papers for citations of other usable studies. This search yields another 11 papers⁶ so that, in total, my sample consists of 515 estimates taken from 29 studies. The studies are listed in Table 1, along with the average estimate of α and selected study characteristics.

⁴ It is worth emphasizing that, by focusing on the parameter value of the Zipf coefficient, the analysis examines only one particular feature of Zipf’s law. Other interesting aspects of Zipf’s law for cities such as the regularity and stability in city size distributions are not discussed.

⁵ I have also experimented with alternative search terms, such as “rank and size and rule” or “Zipf and law.” These variations, however, proved to be less practicable and essentially produced smaller subsets of references.

⁶ In the empirical analysis, I control for this extension.

Table 1
List of studies

#	Author	Year	# Est.	Mean	Std. dev.	Min	Max	Regr. appr. ^a	EconLit
1	Lotka	1925	1	0.930	0.000	0.93	0.93	Lotka	No
2	Singer	1936	23	1.196	0.176	0.93	1.64	Pareto	No
3	Zipf	1949	2	1.011	0.037	0.98	1.04	Lotka	No
4	Allen	1954	69	1.095	0.212	0.70	1.77	Pareto	No
5	Moore	1958	6	0.823	0.109	0.67	0.95	Lotka	No
6	Ward	1963	2	0.950	0.071	0.90	1.00	Lotka	No
7	Lasuen	1967	7	0.750	0.040	0.71	0.80	Lotka	No
8	Lagopoulos	1971	3	0.730	0.030	0.70	0.76	Lotka	No
9	Malecki	1980	45	1.165	0.130	1.00	1.54	Lotka	No
10	Rosen/Resnick	1980	56	1.133	0.192	0.81	1.96	Pareto	Yes
11	Suarez-Villa	1980	1	1.129	0.000	1.13	1.13	Lotka	Yes
12	DeVries	1984	26	0.739	0.089	0.54	0.85	Lotka	No
13	Parr	1985	61	1.147	0.179	0.75	1.64	Pareto	Yes
14	Mills/Becker	1986	58	0.977	0.212	0.49	1.48	Pareto	Yes
15	Alperovich	1989	22	1.147	0.170	0.95	1.61	Pareto	Yes
16	Kamecke	1990	22	1.135	0.188	0.66	1.54	Pareto	Yes
17	Cameron	1990	1	1.006	0.000	1.01	1.01	Pareto	Yes
18	Lepetit	1990	7	0.739	0.013	0.72	0.76	Lotka	Yes
19	Guérin-Pace	1995	24	0.934	0.127	0.70	1.13	Lotka	Yes
20	Krugman	1996	1	1.004	0.000	1.00	1.00	Pareto	Yes
21	Eaton/Eckstein	1997	6	0.984	0.057	0.88	1.03	Pareto	No
22	Black/Henderson	1998	20	0.886	0.044	0.84	0.97	Pareto	Yes
23	Krakover	1998	12	1.371	0.180	1.08	1.86	Lotka	Yes
24	Brakman et al.	1999	3	0.767	0.243	0.55	1.03	Lotka	Yes
25	Gabaix	1999	1	1.005	0.000	1.01	1.01	Pareto	Yes
26	Dobkins/Ioannides	2000	10	0.991	0.029	0.95	1.04	Pareto	Yes
27	Knudsen	2001	1	1.056	0.000	1.06	1.06	Pareto	Yes
28	Song/Zhang	2002	13	1.208	0.183	0.92	1.39	Pareto	Yes
29	Davis/Weinstein	2002	12	1.285	0.314	0.81	1.88	Pareto	Yes

^a Regression approach refers to the regression specification applied in the paper: in the “Lotka” specification, the regressand is the log of city size, while in the “Pareto” specification, the regressand is the log of city rank.

Notes. The summary statistics are based on original Zipf estimates (as reported in the paper). Detailed references are given in Appendix A.

In the statistical analysis, I treat each estimate as a separate observation. This approach risks giving studies with multiple estimates a disproportionate importance and may also introduce heterogeneity bias since estimates from the same paper are not necessarily independent. At the same time, however, it allows me to explore the considerable within-study variation in Zipf estimates. More on this below.

Finally, it is necessary to normalize the Zipf estimates to a common metric. Some authors run Zipf regressions based on Lotka’s [13] specification of the rank–size relationship, where the log of city size is the dependent variable.⁷ These results are transformed by taking the inverse of the estimated coefficient, thereby following Parr [16] and Gabaix and Ioannides [7].

⁷ Accordingly, the estimated coefficient α is the exponent on the city rank.

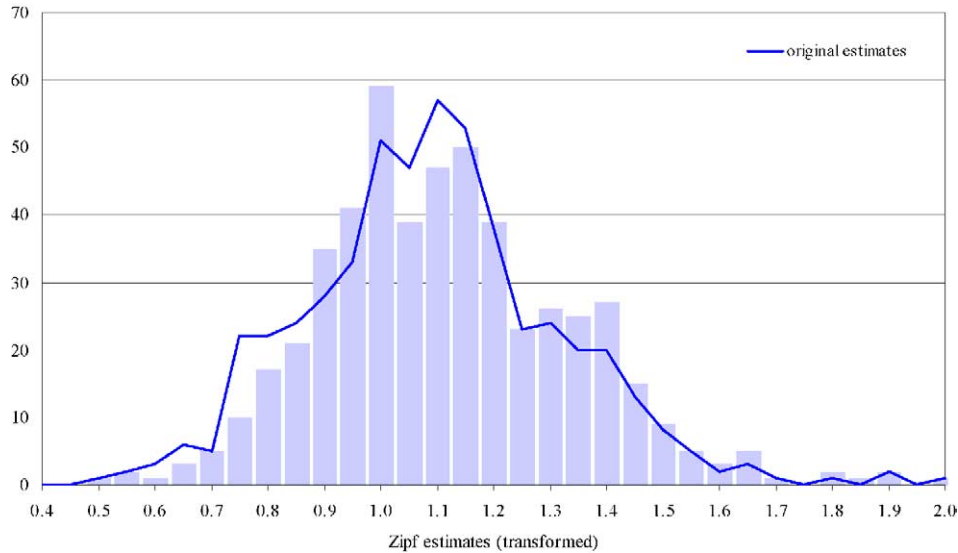


Fig. 1. Histogram of Zipf estimates.

Generally then, the (harmonized) estimates of α in my sample range from 0.49 to 1.96; the mean (median) estimate is 1.09 (1.08). I provide a histogram of the α estimates in Fig. 1. The majority of the point estimates (62%) is larger than 1, and more than one-third ($184/515 = 36\%$) are outside of the interval from 0.8 to 1.2. Fig. 2 shows the Zipf estimates over time (for both the full time period and the sub-period from 1800–2000), with no evidence of a clear time trend.

3. Results

I begin the analysis with a more detailed description of the data. In Table 2, I split the sample along various dimensions and report separate mean values of the estimated Zipf coefficient (as well as a p -value for the equality of means across the different subgroups); the results are tabulated in columns (1) through (4). The top panel of Table 2 focuses on features that are specific to the particular estimation of α ; the second part explores differences in the characteristics of the respective studies.

In a first exercise, I examine the impact of differences in the estimation specification. As shown in the first set of tabulates, the regression approach (i.e., the choice of the dependent variable) has no measurable impact on the results: when the estimates derived from the Lotka specification are transformed, the mean values of α are basically identical for both regression formulations.⁸ This result is reassuring. It suggests that there should be no

⁸ I focus here exclusively on differences in the (standard linear) OLS-based Zipf regression. Recent studies have also increasingly applied more sophisticated econometric techniques (such as nonparametric procedures) to analyze Zipf's law; see, for example, Ioannides and Overman [11].

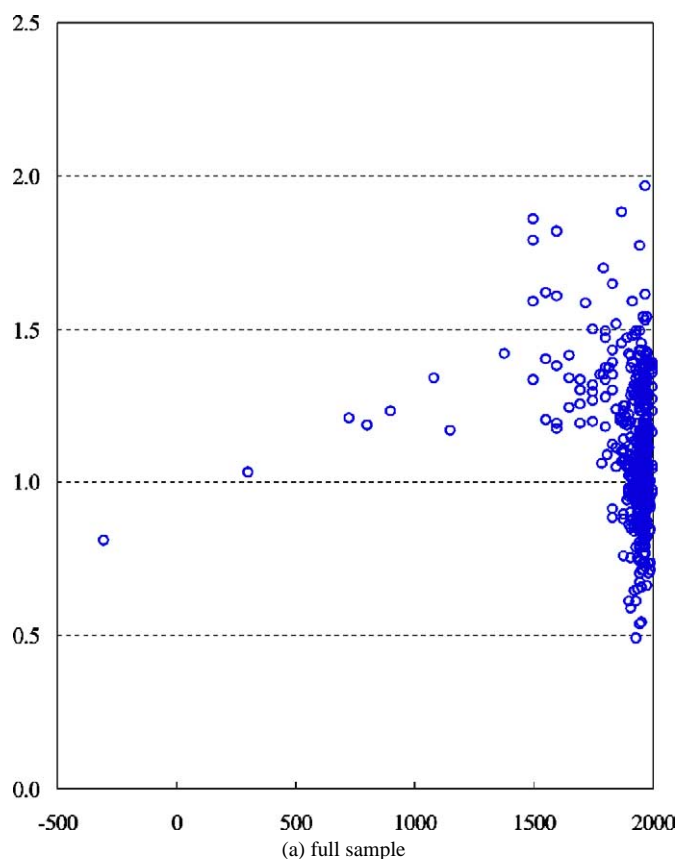


Fig. 2. Estimated Zipf coefficients.

problem in combining the slope coefficients from the two different specifications in the meta-analysis. Moreover, the large majority (73%) of the estimates in my sample is derived from the Pareto form anyway which is, according to Gabaix and Ioannides [7], the preferable regression specification.

Next, I explore whether the definition of an urban area affects the Zipf estimates. A popular argument in the literature on Zipf's law states that the size distribution of agglomerations provides a better approximation to the Pareto distribution than city data. The basic intuition is that the boundaries of cities (in contrast to metropolitan areas) are often administratively defined. By ignoring surrounding suburbs (whose size might vary considerably across locations), the population appears to be more evenly distributed across cities, and the estimate of α is larger. This point was already emphasized by Auerbach [1]; Rosen and Resnick [18] and Soo [20] provide supportive cross-country evidence. The results in Table 2 strongly confirm this hypothesis: for agglomeration data the average Zipf estimate is considerably smaller (and closer to one) than for city data; the difference in means is statistically highly significant.

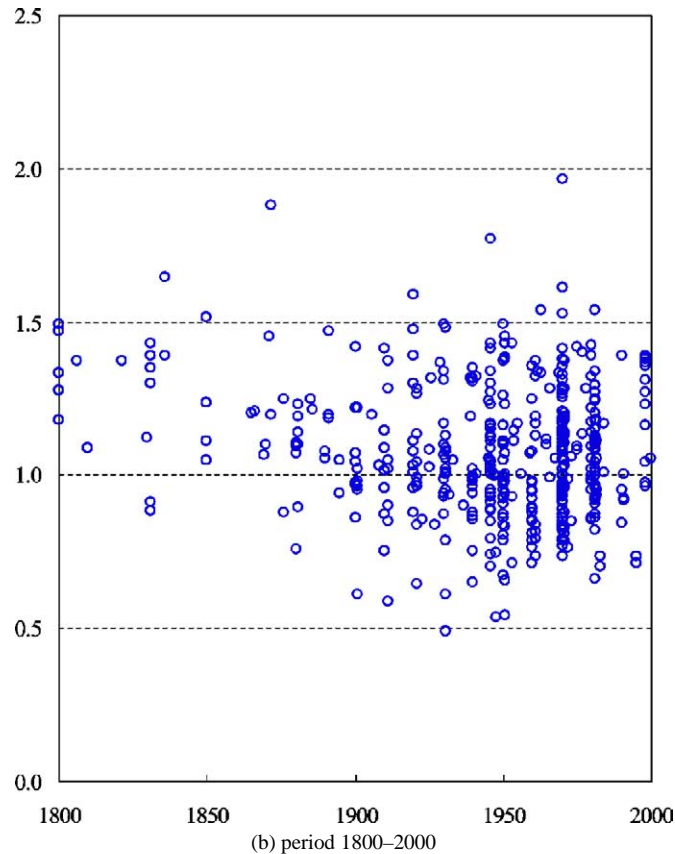


Fig. 2. Continued.

If one distinguishes the estimates of α over time, there appears to be a clear tendency (unobservable from Fig. 2) that the estimates fall over time. Averaging across the period after 1950 produces a mean Zipf coefficient of 1.07, while the mean estimate is 1.35 for distributions before 1801.

Analyses of the city size distribution also span different territorial levels. Dividing the sample along these lines shows that estimates for territories within countries (i.e., regions) tend to be significantly smaller than estimates at the country level, while these themselves are smaller than estimates that comprise more than one country. I also analyze whether estimates of the rank–size distribution in the United States are different from that in other countries. Not surprisingly, it turns out that Zipf estimates for the US are very close to one; the difference to non-US estimates is statistically significant.⁹

⁹ US data are often used to illustrate Zipf's law; examples include Krugman [14], Gabaix [6], and Fujita et al. [5]. Potential explanations for the strikingly good empirical fit of US data are the availability of data for a (reasonably) large number of metropolitan areas and that the US urban system is not dominated by a disproportionately large primate city (possibly due to the federal structure).

Table 2
Bivariate meta-regression analysis

	Descriptive statistics				Bivariate meta-regression						
	Mean	Std. dev.	# Obs.	<i>p</i> -value	Interc.	95% Conf. intv.		Slope coeff.		<i>R</i> ²	
	(1)	(2)	(3)	(4)	(5)	Lower	Higher	(8)	<i>z</i> -stat	Within	Between
All estimates	1.092	0.225	515								
<i>Estimation characteristics</i>											
Dependent variable											
Population	1.096	0.208	376	0.550	1.143	1.055	1.232	−0.053	0.897	0.00	0.03
Rank (transformed)	1.082	0.267	139								
Definition of urban area											
Agglomeration	1.020	0.173	125	0.000	1.165	1.111	1.219	−0.138	4.741	0.03	0.24
City	1.110	0.231	378		1.039	0.977	1.101	0.128	4.361	0.03	0.17
Region	1.285	0.314	12		1.105	1.047	1.164	0.179	1.208	0.00	0.04
Year											
Before 1801	1.352	0.214	42	0.000	1.095	1.047	1.142	0.171	3.738	0.01	0.30
1801–1900	1.185	0.215	54		1.107	1.053	1.160	0.053	1.793	0.00	0.15
1901–1950	1.033	0.211	168		1.127	1.068	1.187	−0.050	2.474	0.01	0.01
After 1950	1.069	0.203	249		1.114	1.056	1.172	−0.003	0.148	0.00	0.12
Unknown	0.998	0.047	2								
Territory											
Region	0.922	0.180	101	0.000	1.127	1.073	1.180	−0.129	3.210	0.01	0.16
Country	1.118	0.208	388		1.029	0.942	1.117	0.098	2.463	0.01	0.03
Continent	1.375	0.186	26		1.101	1.044	1.157	0.274	1.995	0.00	0.10
Country											
USA	1.040	0.139	85	0.019	1.133	1.075	1.191	−0.079	2.359	0.01	0.09
Other	1.103	0.237	430								
Data source											
National statistical office	1.051	0.201	261	0.000	1.109	1.048	1.170	0.006	0.287	0.00	0.12
UN demographic yearbook	1.102	0.206	80		1.114	1.055	1.174	−0.025	0.676	0.00	0.00
Other	1.149	0.254	174								
Cut-off criterium											
Population size	1.090	0.222	316	0.072	1.103	1.035	1.172	0.016	0.500	0.00	0.00
Number of cities	1.127	0.180	112		1.117	1.056	1.179	−0.019	0.524	0.00	0.00
Unknown	1.054	0.278	87								
Number of observations											
Less than 21	0.896	0.281	40	0.000	1.121	1.062	1.180	−0.155	4.636	0.04	0.00
21–100	1.107	0.225	163		1.086	1.024	1.147	0.066	2.810	0.02	0.01
101–1000	1.081	0.165	109		1.106	1.048	1.165	0.022	0.904	0.00	0.06
More than 1000	1.045	0.194	49		1.113	1.055	1.170	−0.001	0.014	0.00	0.04
Unknown	1.151	0.226	154								

(continued on next page)

Table 2 (Continued)

	Descriptive statistics				Bivariate meta-regression						
	Mean	Std. dev.	# Obs.	<i>p</i> -value	Interc.	95% Conf. intv.		Slope coeff.		<i>R</i> ²	
	(1)	(2)	(3)	(4)	(5)	Lower (6)	Higher (7)	(8)	<i>z</i> -stat (9)	Within (10)	Between (11)
<i>Study characteristics</i>											
Year of publication											
Before 1950	1.176	0.176	26	0.053	1.113	1.051	1.174	−0.002	0.017	0.00	0.00
After 1950	1.088	0.227	489								
Econlit											
Yes	1.085	0.218	325	0.355	1.149	1.056	1.242	−0.059	0.973	0.00	0.03
No	1.104	0.237	190								
Number of estimates											
1	1.005	0.066	6	0.011	1.124	1.066	1.183	−0.119	1.244	0.00	0.09
2–10	1.183	0.206	46		1.080	1.015	1.144	0.113	1.848	0.00	0.12
More than 10	1.084	0.226	463		1.144	1.051	1.234	−0.053	0.861	0.00	0.01
Analyzed period											
Less than 11 years	1.134	0.182	123	0.000	1.124	1.055	1.193	−0.036	0.584	0.00	0.05
11–50 years	0.868	0.208	60		1.133	1.073	1.194	−0.182	2.003	0.00	0.05
More than 50 years	1.117	0.186	332		1.052	0.970	1.134	0.106	1.920	0.00	0.14
Analyzed territories											
1	1.039	0.231	186	0.000	1.137	1.045	1.228	−0.041	0.671	0.00	0.02
2–10	1.119	0.249	121		1.103	1.037	1.170	0.041	0.574	0.00	0.01
More than 10	1.125	0.195	208		1.109	1.045	1.174	0.018	0.228	0.00	0.00
Modifications (deviations from standard specification)											
Yes	1.120	0.293	88	0.198	1.116	1.056	1.176	−0.024	0.538	0.00	0.02
No	1.086	0.208	427								
Sensitivity (experimenting with different specifications)											
Yes	1.059	0.213	405	0.000	1.167	1.057	1.276	−0.076	1.149	0.00	0.00
No	1.213	0.228	110								

The results are harder to interpret for differences in the data source: estimates derived from data obtained from national statistical sources appear to be considerably smaller than estimates based on other data sources. This may reflect better data availability; it has often been argued that a larger number of observations (i.e., including the lower tail of the distribution) produces lower Zipf coefficients.¹⁰ However, differentiating across the number

¹⁰ Gabaix and Ioannides [7, fn. 4] note: “The exponent [...] is sensitive to the choice of the cutoff size above which one selects the cities. For a lower cutoff, the exponent is typically lower.”

of observations yields no clear tendency. A possible explanation for this divergence from individual study findings is that, given the large variety in data sets, the number of observations is only an imperfect indicator of the analyzed distribution tail.¹¹ There is also little evidence that the cut-off criterium has a measurable effect on the Zipf estimates.¹²

Turning to study characteristics, there are two notable results. First, EconLit coverage does not affect the Zipf estimate. This is reassuring; my extension of the selection procedure appears to have no (distortionary) effect on the results. Second, there is strong evidence that an increase in the number of estimates per study is associated with larger deviations from Zipf's law. Most notably, if a study reports only a single estimate, the mean value of the Zipf coefficient is almost exactly 1. This is not surprising; these studies typically report only an illustrative example, whereas studies that report multiple estimates often perform extensive sensitivity analyses, yielding a larger range of coefficients. However, if one explicitly controls for studies that modify the standard regression approach (e.g., by excluding the largest city from the sample) (*modifications*) or experiment with alternative regression specifications (*sensitivity*), the results are inconclusive.

While descriptive statistics provide a useful summary of the empirical evidence on Zipf's law, only meta-analysis allows to identify the precise impact of estimation characteristics on the variation in estimated Zipf coefficients. In a next step, therefore, I regress the point estimates of α on features of the underlying study. In particular, I estimate variants of the equation:

$$\hat{\alpha}_{ij} = \beta + \gamma X_{ij} + \varepsilon_{ij},$$

where $\hat{\alpha}_{ij}$ is the i th Zipf estimate of the j th study, X_{ij} is a set of meta-independent (moderator) variables, and ε_{ij} is an independently distributed error term; β and γ are parameters to be estimated.

Although results from the same study are not necessarily independent, I treat each available Zipf estimate as a separate observation. This implies that studies with multiple estimates enter the regression with a disproportionately large weight.¹³ To deal with this issue, I follow standard procedures and estimate a panel specification with random (study) effects (see, for instance, Jeppesen et al. [12]); fixed effects estimates were marginally weaker. I also experimented with subgroup analysis, but none of the results changes substantively when studies are omitted one by one.¹⁴

¹¹ Rosen and Resnick [18] report equally inconsistent results. They find that Zipf estimates are strongly affected by sample size but that the direction and the magnitude of the effect varies. Also the results in Malecki's [15] and Guérin-Pace's [9] detailed country studies are rather inconclusive.

¹² There are three possible criteria to decide (in a consistent manner) where to cut-off the sample of cities included in the analysis: a fixed number of cities, a fixed (absolute) population size of a city, or a size above which the sample accounts for some fixed proportion of a country's population. See Cheshire [3] for a discussion.

¹³ There is practically no alternative to this approach. Reducing the results from each study to a single observation would not only be difficult, but ignore most of the relevant information (e.g., the within-study variation in large cross-country samples or the results from sensitivity analyses).

¹⁴ It should also be noted that my main interest is the magnitude of the Zipf estimate and not its precision. A popular type of meta-analysis aims to explore the statistical significance of an effect. Similarly, there is no reason to examine the sample for possible publication bias.

I begin with a simple bivariate meta-regression analysis; that is, I perform a separate regression for each moderator variable. The results are reported in columns (5) through (11) of Table 2. The two main parameters of interest are the intercept, tabulated in column (5), and the slope coefficient, reported in column (8). The intercept term gives the (random effects) point estimate of Zipf coefficient α for studies that do not show the analyzed characteristic, while the slope coefficient captures the extent to which estimates with this particular feature differ from the rest of the sample.

As shown, pooled across studies, the Zipf estimate usually takes values considerably above one. In all but three (of the 33) cases, also the lower bound of the 95% confidence interval exceeds the value of one. This result implies that the strict version of Zipf's law (i.e., $\alpha = 1$) does not hold empirically; it also questions the looser interpretation of Zipf's law that the parameter α is typically in a range that is centered around one (i.e. $\alpha \in [0.8, 1.2]$).

The slope coefficients basically confirm the results from the descriptive analysis. Zipf estimates derived from agglomeration data tend to be significantly smaller than estimates based on city or region population data. Similarly, city sizes appear to be less evenly distributed (and Zipf coefficients are smaller) for territories below country-level (i.e., regions). Finally, there is again evidence that the estimates of α fall over time; estimates for years before 1801 appear to be comparably large.

Table 3 shows the results of multivariate meta-regression analysis. The first column tabulates the benchmark regression for a small set of standard moderator variables. In particular, I control for features of the meta-sample selection procedure and examine the impact of the two main data issues in Zipf regressions: the definition of city boundaries and the number of included observations. The analysis basically replicates previous results. I confirm that the choice of the regression specification (if properly transformed) has no effect on the results; also the coefficient on a dummy variable for EconLit coverage is still not significantly different from zero. More importantly, however, the results concerning the two main hypotheses about estimated Zipf coefficients are again clear and convincing: while there is support for the hypothesis that Zipf regressions based on agglomeration data produce smaller estimates of α than those using other data, there is little indication that an increase in the number of observations is associated with smaller estimates. On the contrary, the coefficient on a dummy variable for 20 or less observations is negative and statistically significant, indicating that Zipf estimates derived from small samples (of potentially large cities) are unusually low. This finding questions, for instance, Gabaix's [6] suggestion that small cities tend to have smaller local Zipf coefficients.

The inclusion of other explanatory variables, while leaving the main results largely unaffected, yields some additional insights. For example, the strongest impact on the estimated Zipf coefficient (both in absolute size and economic significance) results from historical data; Zipf estimates for city size distributions before 1800 are significantly larger. Of the study characteristics, the analyzed time span appears to matter, with studies covering 11 to 50 years reporting on average smaller Zipf estimates than studies covering longer or shorter time periods; a pattern that is hard to explain. Also if a paper reports only a single point estimate of α , this estimate tends to be significantly smaller.

Table 3
Multivariate meta-regression analysis

		Benchmark		Estimation characteristics		Study characteristics		All	
		Coeff.	z-stat.	Coeff.	z-stat.	Coeff.	z-stat.	Coeff.	z-stat.
Intercept		1.161	21.330	1.192	7.234	1.353	10.820	1.159	7.705
<i>Estimation characteristics</i>									
Depend. variable	Population	−0.012	0.215	0.020	0.419	−0.044	0.514	0.031	0.630
Def. urban area	Agglomeration	−0.126	4.355	−0.074	0.605			−0.076	0.693
	City			0.027	0.228			0.045	0.430
Year	Before 1801			0.188	3.346			0.188	3.398
	1801–1900			0.083	2.437			0.084	2.391
	1901–1950			−0.003	0.109			−0.004	0.146
Territory	Region			−0.040	0.348			−0.008	0.078
	Country	0.070	1.800	−0.133	1.111			−0.067	0.681
Country	USA	−0.073	2.263	−0.083	2.511			−0.072	2.303
Data source	Nat. stat. office			−0.009	0.346			0.001	0.054
	UN demogr. y'bk.			0.043	1.024			0.071	1.766
Cut-off criterium	Population size			−0.023	0.491			−0.033	0.793
	Number of cities			−0.006	0.113			−0.021	0.450
Number observat.	Less than 21	−0.144	4.311	−0.144	3.314			−0.142	3.137
	21–100			0.008	0.233			−0.010	0.282
	101–1000			0.026	0.817			0.005	0.154
<i>Study characteristics</i>									
Year publication	Before 1950					−0.048	0.380	0.002	0.029
Econlit	Yes	−0.041	0.699	−0.044	0.873	−0.093	0.982	−0.064	1.829
Number estimates	1					−0.259	1.770	−0.226	2.358
	2–10					0.041	0.485	0.122	2.870
Analyzed period	Less than 11 y.					0.000	0.003	0.138	3.816
	11–50 years					−0.253	2.019	−0.194	3.368
Analyz. territories	1					0.026	0.248	0.082	1.792
	2–10					−0.046	0.408	0.073	1.267
Modifications	Yes					−0.048	0.977	−0.052	1.085
Sensitivity	Yes					−0.151	1.744	−0.108	2.526
No. of obs.			515		515		515		515
R^2 within			0.080		0.100		0.000		0.100
	between		0.330		0.550		0.360		0.620

4. Concluding remarks

This paper applies meta-analytic techniques to summarize the empirical evidence on Zipf's law for cities. The meta-analysis combines 515 estimates from 29 studies, covering a wide range of different territories and time periods.

The main finding of this quantitative review of the literature is that the estimated exponent in a Zipf regression is on average not 1.0. If the regression is properly specified in the Pareto form, the pooled estimate of α is considerably larger than one, close to 1.1. This finding is remarkably similar to results from large cross-country studies by Rosen and Resnick (with an average Zipf estimate of 1.13) and Soo (1.11); it questions (the strict form of) Zipf's law for cities.

In addition, I identify some design features of individual studies and estimates that help to explain the large variation in α estimates across studies. In particular, I find that point estimates of α are significantly smaller if the estimate is based on population data for metropolitan areas (instead of inner cities), the estimate is based on data for recent years, the estimate is for the US city size distribution, the sample comprises only a small number of observations, and the study reports only a single estimate.

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Appendix A. List of studies included in the meta-analysis

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